White Paper on Pricing Algorithms

Derived Data, Inc.

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Pricing Algorithms

Introduction

Most fixed income instruments trade OTC (Over-The-Counter) and unlike equities there are no settlement or official closing prices for them. The average daily traded notional value of all fixed income instruments and their derivatives, including currencies and swaps is about 50 times larger than the market value of traded equities. Likewise, the number of fixed income securities is significantly larger than the number of listed equity tickers, since a listed equity can have many bonds with different maturities and ratings.

Fixed income securities are highly correlated and securities of an issuer along with its derivatives are very tightly linked with each other through mathematical relationships. These relationships can be used to calculate the fair price of an instrument from the traded prices of other instruments of an issuer. The most popular pricing algorithms are known as matrix or reference pricing. Matrix pricing algorithms usually use one or two reference securities with similar rating, maturity or duration profile to price another security. Obviously, if the price of the reference security is not accurate or if the implied rating of a security is different from the stated rating, this pricing methodology fails. Most of the time the market participants anticipate rating changes and trade bonds as if the rating change has been announced. There have been several articles in the press that have criticized matrix pricing methodologies[1].

The development of TRACE (Trade Reporting and Compliance Engine) mandated by SEC (Securities and Exchange Commission) and managed by FINRA (Financial Industry Regulatory Authority) has increased transparency in corporate bond market trading. However, the resulting transparency has not increased liquidity in a material way and most small issues trade with relatively large pricing variations. Some issues may not trade for weeks at a time, while fund managers need accurate valuation and pricing for all their holdings on a daily basis.

Our pricing methodology is based on the Term Structure of Rates (TSR) which has been extensively covered in the book "The Advanced Fixed Income and Derivatives Management Guide" (AFIDMG), published by John Wiley and Sons. We calculate TSR for all global treasuries, real rates, Libor (Swaps) and global corporate rates for all issuers, where the data is available. As such, our pricing algorithms depend on the aggregate of the traded securities of an issuer, rather than one or two reference securities. Additionally we perform both cross-sectional (Comparison of all securities on a daily basis) and time-series analysis, in order to make sure that inconsistent data is filtered out.

We probably have the most robust and accurate methodology for pricing derivatives and callable bonds. In order to price callable bonds when the bond price is close to the call price, a very extensive analysis of the value of the call needs to be performed; matrix pricing can have a significant pricing error for callable securities.

[1] http://www.ft.com/cms/s/0/e264f1e4-1ef1-11e4-ad93-00144feabdc0.html#axzz4CB2qxBrH

Term Structure of Rates (TSR)

Term Structure of Rates (TSR) is the relationship between the yields (rates) and maturities (terms) of an issuer. For accuracy and consistency, the TSR uses spot or zero coupon yields of an issuer. Even though most securities are not zero coupon bonds, a zero coupon curve can be calculated in such a way that if all cash flows of a bond of an issuer are discounted by the implied TSR yields, the calculated price of the bond will be equal to its market price. The accuracy of TSR to price bonds of an issuer is dependent on the bid-ask spreads of bond prices and the quality of the data. We can use the TSR to price all bonds of an issuer with an average accuracy of less than bid-ask spread of their market price. For US treasuries, the price spread is usually about one tick (1/32) while for medium duration high yield bonds it can be half a point. The TSR is a representation of the discount function that the market uses to price the securities of an issuer and has been extensively covered in AFIDMG.

TSR is a a snapshot of the yield curve of all bonds of an issuer and is a very powerful method for capturing pricing errors. Once the TSR of an issuer is calculated from its traded bonds, untraded or unpriced securities of the issuer can be priced from the TSR. Due to liquidity, market segmentation or other factors, bonds of an issuer can have a yield that is slightly blelow or above their TSR levels; the yield difference is called spread to the curve. Spread to the curve tends to be very stable and it provides a very efficient method for pricing untraded securities. Likewise, if the calculated price of a bond is significantly different from its market price feed, it is usually because the feed price is incorrect.

There are no reference securities in TSR, the reference is the entire market; if a security has a bad price, its price would not be propagated to other securities if the bad price was from a reference security. In fact, if the price of the reference security that is used by other price vendors is bad, we can usually filter it out completely and can calculate a more robust price for it.

For US and other liquid global treasuries and Libor we use 5 components for our TSR. The first component or Level of rates is a measure of the general level of rates of an issuer. The second component or Slope is a measure of the steepness or inversion of the yield curve. The first two components account of more than 95% of the changes in the US treasuries. The third component or Bend is a measureof the hump or curvature of the curve. The bend usually account for the relative performance of the five year part of the curve relative to the short and long rates. The first three components account for nearly 99% of the movements of the US treasury rates.

We use the same TSR methodology for the spread of corporate and emerging markets bonds. The level of the spread is a mearure of the credit worthiness of an issuer; it respresents a curve that is perfectly parallel to the treasury curve and except for its level, all other components are identical to the treasury curve. The slope of the spread is a measure of the steepness of the credit curve relative to the treasuries. A steep credit spread imply increasing credit risk and an inverted spread curve imply decreasing credit risk of an issuer. Usually, high yield securities have an inverted spread curve and high grade bonds have a steep credit spread curve. The bend or hump of the credit spread is a measure of an inflection in the riskiness of an issuer; it can signify an implied improvement and a subsequent decline or vice versa for the creditworthiness of an issuer. We use a maximum of three components for the term structure of credit spreads and that is a significant improvement over traditional methods of just using a single spread.

Cross Sectional and Time Series Analysis

Our cross-sectional analysis involves analysis of the TSR of all issuers to make sure that they are realistic and accurate. For example, if the TSR is very steep in such a way that one year rates are 1% and 2 year rates are at 5%, it would imply that one year from now the one year rates would be 7%. Such a large rise in one year rates has never happened and market participants being aware of the opportunity, simply buy two year rates and sell one year rate until the scenario becomes more realisitic. We use historical ranges to limit the components of the TSR for treasuries and corporates. For example, we limit the bend component to a range of ±75 bps, since in the liquid markets of US and other global treasuries it has never exceeded these limits, since 1980. Likewise, we limit the maximum slope of the term structure of credit spread to no more than 2%. If the calculated components of the TSR exceed these levels, we force them to these levels.

We also perform cross-sectional analysis on the yield of all the bonds of an issuer that have similar seniority. For every bond, we calculate the out-of-sample yield and its standard deviation. If the yield deviates by more than 4 sigma from the out of sample average, we flag the price of that bond.

The time series analysis involves comparing the historical spreads of a security relative to its curve, where the data is available, for two months. We measure the ratio of the spread to its historical standard deviation; if it is more than 3 standard deviations from the mean (99.5%), we assume that it is a pricing error and we thus calculate the price for that security from its spread. Since spread analysis is done relative to the curve of the issuer, time series analysis captures pricing errors better than any other methodology.

Time series analysis is also performed on the price change of a security versus the standard deviation of its historical changes. If the price move is more than 3 sigma, we then look at other bonds of the same issuer. If they had similar moves, the price move is valid, otherwise we flag the price and assume that it may not be accurate.

Time series analysis is also very useful in identifying bonds whose price don’t change. If the price of a bond stays constant for 3 days in a row but other bonds of the same issuer have price changes, we assume that the price is stale and calculate a price for that bond.

Intraday and After Hour Adjustments

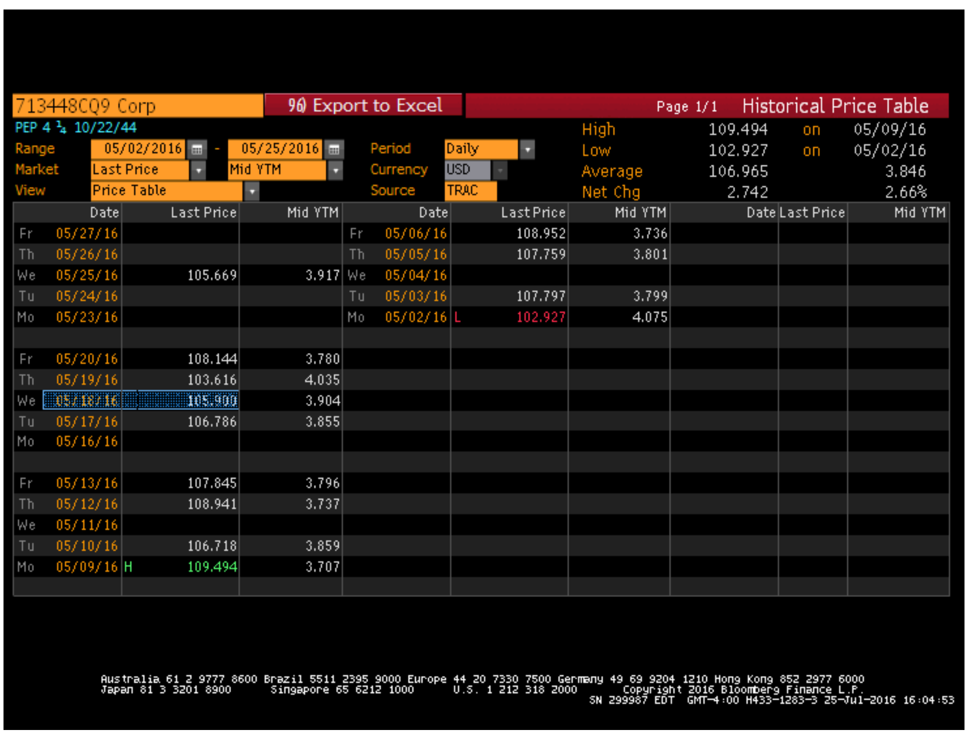
Most US treasuries are very liquid and there is an active market for them on a continuous basis. Thus, we can calculate the TSR for US treasuries at all times during the tading day. However, many corporate bonds do not trade every day and when they do, the trade time can be different from the closing time. The level of interest rates could be significantly different at the time that a corporate bond trades than the conventional closing time (3:00 PM Eastern Time). For example, a 30 year corporate bond with a duration of 15 years can trade at 8:00 Eastern time at a price of 101. Given a rise of 8 basis points from the time the corporate bond trades through the end of the day, due to a strong payroll number, the impact on the corporate bond price would be 0.08*15=1.2%. If the corporate bond does not trade again during the day, most pricing services will provide a closing price of 101, even though the level of rates has changed significantly. However, we calculate the US TSR on a regular basis during the trading day and adjust the prices of US corporates based on the time of the day that they traded; in this case our price will be 101*(1.-1.2%)=99.788.

Figure 1 shows the intraday yield of the on-the-run 30 year US treasury on May 18, 2016. From the time the corporate bond market starts trading till closing time, the yield rose about 9 basis points.

Figure 1. Intraday yield of 30 year Treasury – May 18, 2016

Let us examine a long maturity corporate bond, e.g, PEP 4.25%, 10/22/44 with a duration of about 16.5 years. This bond traded at 8:01 AM at a price of 105.9. At that time the yield of the 30 year treasury was 2.596%. By the close of the market, the yield of the 30 year treasury had risen by 9 bps to 2.687%, but the corporate bond did not trade anymore on that day. Given the rise in yields, the price of the corporate bond would have to adjusted downward by -16.5*0.09=-1.485%. Thus, the closing price would have to be 104.33. We make such an adjustment to the price of the bond as part of our process, however, most pricing services and Bloomberg do not. Figure 2 is a screenshot of the traded time of our corporate bond and Figure 3 is a table of its historical closing price, with the shaded cells corresponding to May 18, 2016.

Figure 2. Intraday price of PEP 4.25% 10/22/44– May 18, 2016



	Date	Last Price	Mid YTM		Date	Last Price	Mid YTM
Fr	05/27/16			Fr	05/06/16	108.952	3.736
Th	05/26/16			Th	05/05/16	107.759	3.801
We	05/25/16	105.669	3.917	We	05/04/16		
Tu	05/24/16			Tu	05/03/16	107.797	3.799
Mo	05/23/16			Mo	05/02/16 L	102.927	4.075
Fr	05/20/16	108.144	3.780				
Th	05/19/16	103.616	4.035				
We	05/18/16	105.900	3.904				
Tu	05/17/16	106.786	3.855				
Mo	05/16/16						
Fr	05/13/16	107.845	3.796				
Th	05/12/16	108.941	3.737				
We	05/11/16						
Tu	05/10/16	106.718	3.859				
Mo	05/09/16 H	109.494	3.707				

Figure 3. Historical price of PEP 4.25% 10/22/44 - May 18,2016

Trading in the US usually continues past the 3:00 PM cutoff time and due to the globalization of trading, many securities trade around the clock. We capture any other trading that takes place after the 3:00 PM and use that for future analysis and for calculating the spreads of bonds that don't trade every day.

Summary

Our pricing process involves the following steps:

1. We obtain prices and perform intraday adjustments
2. Historical time series analysis of the price moves is performed and bonds with suspicious prices are assumed not to have been priced.
3. The Term Structure of all rates are calculated and rescaled to correct for prior day's bonds prices (See *Non-Callable Credit Bonds* Section in the Appendix).
4. Unpriced securities are priced.
5. New Term Structure of all rates are calculated.
6. All durations, spreads and call values are recalculated including bonds that were flagged in Step 2, using their quoted market prices.
7. Time series and cross-sectional analysis is performed as follows:
    a. Time series analysis on prices, similar to Step 2.
    b. Time series analysis of spreads relative to the respective curves.
    c. Cross-sectional analysis of yields and comparison with out-of-sample averages and standard deviations.
    d. Analysis of the daily changes in the spread of bonds relative to the issuer's curve.
8. New prices for flagged securities are calculated. A bad price is usually flagged by more than one of the 4 above methods.
9. Steps 5 and 6 are repeated with new prices.

It should be noted that a price maybe flagged by one method but not by others. For example, if the price of a bond that has only 3 months to maturity, changes by 0.25%, its spread changes by 100 bps, resulting in a flag in spread time series analysis but not in price analysis.

Our adjustment process employs a soft barrier methodology. For example, we don't completely revise a price that is 3 sigma away from mean, but leave a 2.99 sigma event unchanged. We use a continuous method of deriving new prices that is a function of the price deviation from mean. As such, the price of all bonds are revised, even US treasuries. However, the revision for a treasury bond price can be in the 3rd or 4th decimal place, but the revision for a corporate bond price can be several points.

Appendex A

US Treasuries

US treasuries are the most liquid of all fixed income instruments. Even though they trade primarily OTC, partly due to their size and partly due to the exchange traded futures which closely track cash bonds, they tend to have a bid-ask spread of about one tick (1/32) for liquid securities and yield spread of about one basis point for most Strips. Figure A-1 shows the TSR for US treasuries.

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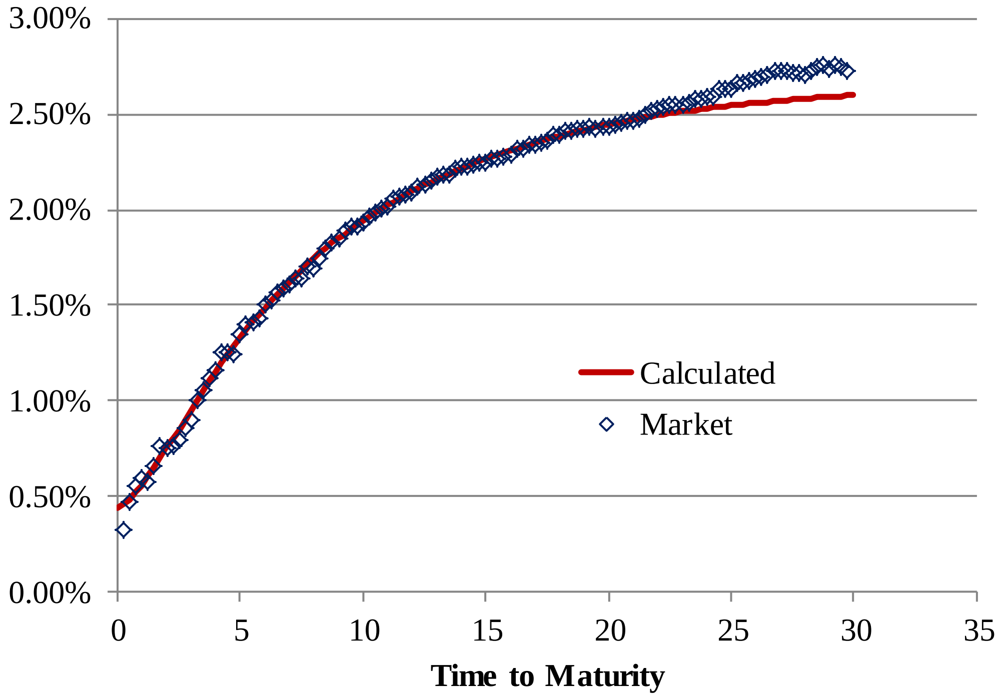

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Figure A-1. Term Structure of Rates and Market Prices 2/12/2016

The calculated curve is the spot curve and is comparable to the treasury coupon Strips. The standard deviation of the difference between the calculated yield of treasury Strips and the market yield is less than one basis point up to a maturity of more than 20 years and the resulting pricing error if we use the curve for it, is less than the traded bid-ask spread. Longer dated Strips have a positive spread to the curve, but the spread will not affect the pricing accuracy as will be explained shortly.

Figures A-2 and A-3 show the time plot of the continuously compounded yield and spread relative to the TSR of the 20 year treasury respectively. The range of spread of the security relative to the curve is less than 2 basis points, even though, this security was not used to calculate the curve. The daily variation of the spread is less than 0.25 basis points with a strong mean reversion tendency.

Figure A-2. Yield of 20 Year Treasury

Figure A-3. Spread of the 20 Year Treasury Relative to the TSY

The stability of the spread of a security relative to its curve is the basis of our pricing methodology. The TSR is a representation of the entire market of an issuer. Most market participants are aware of the relative value of different securities and if a security becomes too cheap or rich relative to its peers, traders take advantage of the pricing opportunity and bring the spread in line with the curve. Other factors such as liquidity, size of an issue and coupon rate all contribute to the desirability or lack thereof of a security and result in a market consensus stable spread for a security. The spread can change from one day to the next, but its changes are generally smaller than transaction costs and are highly mean reverting.

Matrix pricing uses similar reasoning for pricing, but the pricing error tends to be about 50% higher than curve pricing, since the matrix or reference security is subject to price variation. Reference securities themselves tend to have higher spread volatility due to the liquidity premium at the time of issuance. Figure A-4 shows the spread of the 10 year treasury issued on 2/15/2015. The security had a spread of -14.5 bps at the time of issue and over the following six months the spread changed by 6 bps; compare the range in Figure A-4 that to Figure A-3.

Figure A-4 - Spread of the 10 Year Treasury Relative to the TSR

The spread volatility of reference or benchmark securities can result in a significantly larger pricing error at times of crisis when liquidity premium can be very volatile and large, such as the period post Lehman bankruptcy.

Global Treasuries

Most global government bonds have very good liquidity and many are exchange traded. We calculate a TSR for all global governments and real rates where the data is available. When an illiquid bond trades, we capture its spread relative to the respective curve and use that spread to price the security on other dates. If no spread is available, we assume that the spread is zero.

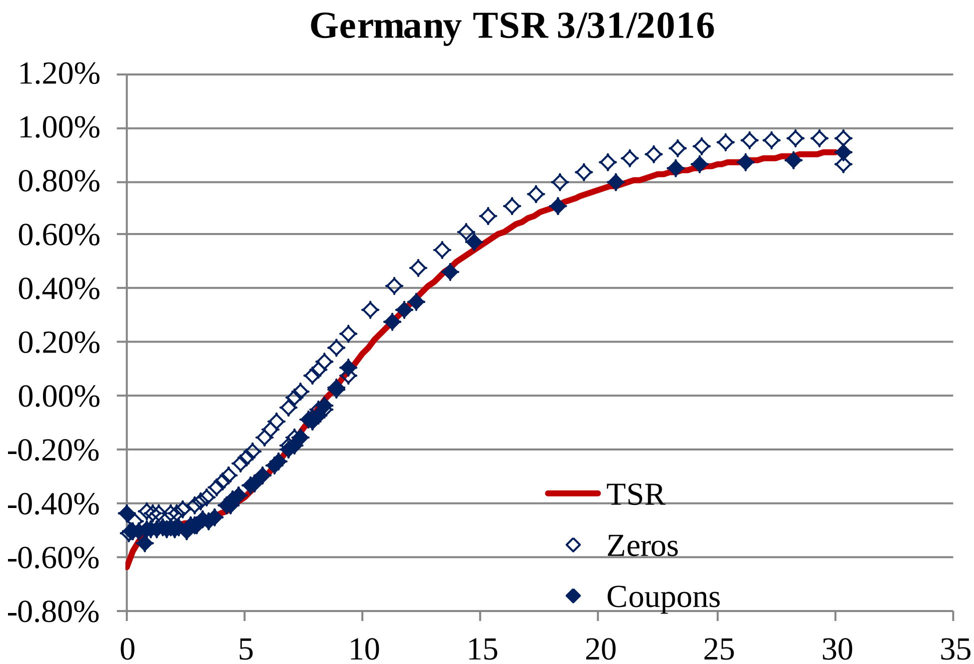


Figure A-5 - German Government Curve and Traded Securities

Figure A-5 shows the TSR of German government bonds including zero coupon bonds. For coupon bonds we calculate their spread relative to the curve and simply add that spread to the spot curve (TSR) for visual purposes. Solid diamonds represent the coupon bonds and were used to calculate the TSR. Blank diamonds are based on the market price of zero coupon bonds. Here is a very good case of market segmentation; traders prefer coupon bonds and are willing to pay a yield premium of 15 bps over zero coupon bonds. A portfolio manager can simply replicate a coupon bond by buying a stream of zero coupon bonds. Had we built our TSR from zero coupon bonds, coupon bonds would fall under the TSR. Table A-1 shows some of these bonds

Table A-1 - German Government Bond Prices and Spreads

Coupon	Maturity	Price	Spread
0	2/15/2023	100.29	0.153%
0	5/15/2023	100.06	0.158%
0	8/15/2023	99.88	0.155%
1.5	2/15/2023	111.84	-0.008%
1.5	5/15/2023	112.12	-0.015%
2	8/15/2023	116.07	-0.016%

Figure **A-5** shows the uniformity of the spread of bonds above the curve or right on it. The uniformity is also a sign of stability of the spread and the spread can be used to price bonds that are not traded.

Non-Callable Credit Bonds

Our methodology for pricing corporate bonds and agencies is similar to pricing treasuries. However, market depth (size and liquidity) and breadth (number of maturities) of corporate bonds is significantly lower than government bonds. Most corporate bonds have outstanding amounts of about $250 million or less. Assuming a very conservative average position size of $2.5 million for each fund manager, the number of holders for a $250 million issue will be about 100. If the average holding period is one year, the expected number of trades will be about twice a week. For this reason, many corporate bonds don't trade every day, but need to be priced for fund managers that have them in their portfolios.

Consider a 10 year credit security with a spread of 150 bps over the US treasury curve (Figure A-1). Now assume that the same issuer has a 20 year bond with a spread of 200 bps. From these two bonds we can estimate that if the issuer has a 5 year bond, its spread should be less than 150 basis points. We take this concept and develop a Term Structure of Credit Spread (TSCS) from the traded credit securities in the market. We use 5 components of the TSR for calculating government curves, but we limit the number of components to 3 for credit securities. Even for agency bonds which can be more liquid than many government bonds, three components is sufficient.

Given the liquidity and breadth of agency bonds such as FNMA or FHLM, their credit spread can be fully explained by three components of our TSCS to an accuracy of about 2-3 bps.

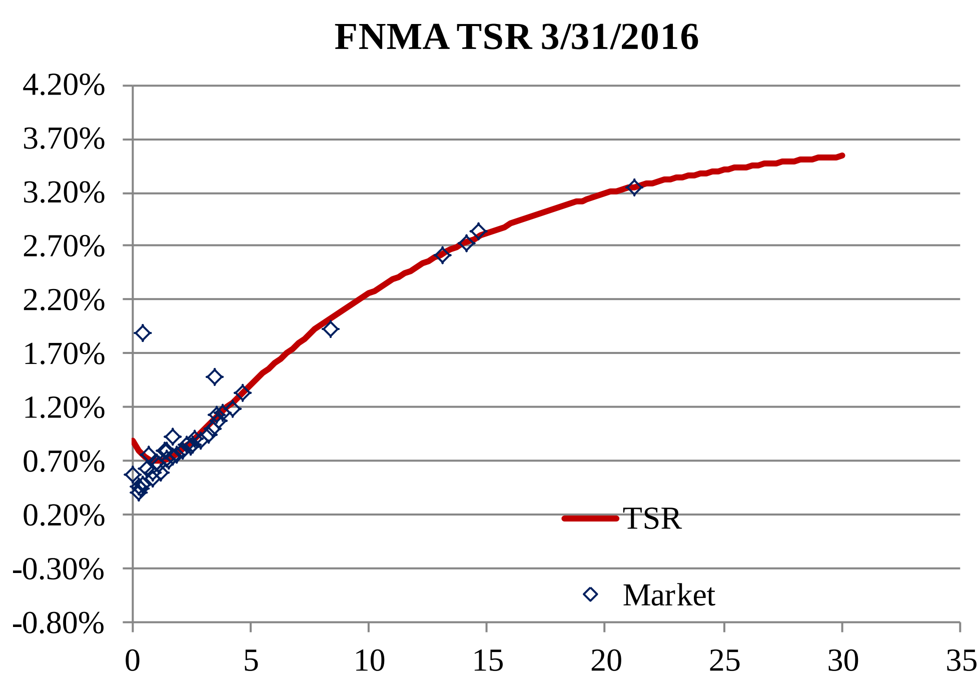


Figure A-6 - FNMA Curve and Traded Securities Using 3 Components

Now, consider a corporate issuer with 10 outstanding bonds and suppose that only 6 of those bonds trade on a given day. We calculate a TSR for that issuer using 6 bonds and use a maximum of 3 components to price them. On the following day, if 5 bonds of the issuer trade, we will calculate a TSR for those 5 bonds. Now assume that only 3 of the 5 bonds traded on the previous day. We adjust the level of the TSR on the second day in such a way that the 3 bonds that traded on both days, are on average correctly priced. This step is a critical component of our pricing methodology and ensures that the aggregate traded bonds (not individual reference securities) have market prices. Once we adjust the TSR, we can price the two bonds that were traded the day before from their spreads to the curve.

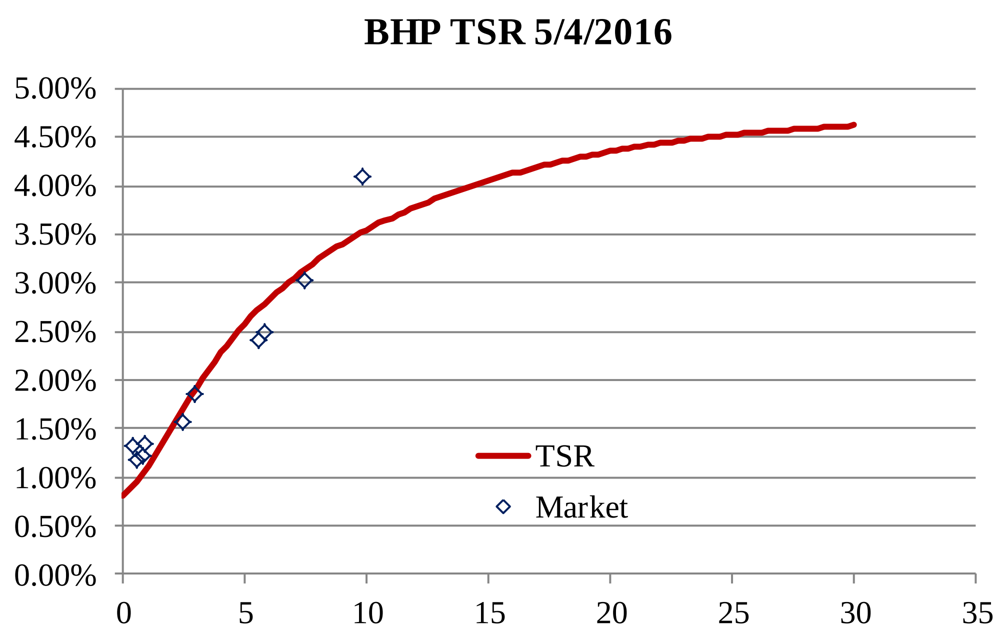


Figure A-7 - BHP Curve and Traded Securities Using 3 Components; 5/4/2016

Figure A-7 shows the TSR for BHP on 5/4/2016 along with traded bonds. The fit is not very good and one of the bonds with a maturity of 10 years has a spread to the curve of more than 50 bps.

On 5/5/2016 the bond with a maturity of 10 years did not trade, but two bonds with maturities of longer than 25 years traded. The calculated TSR for 5/5/2016 is shown in Figure A-8. The fit is significantly better than the previous day. We can speculate that the bond with a maturity of 10 years is an old legacy bond that was issued in 1996, is not liquid and has a large spread.

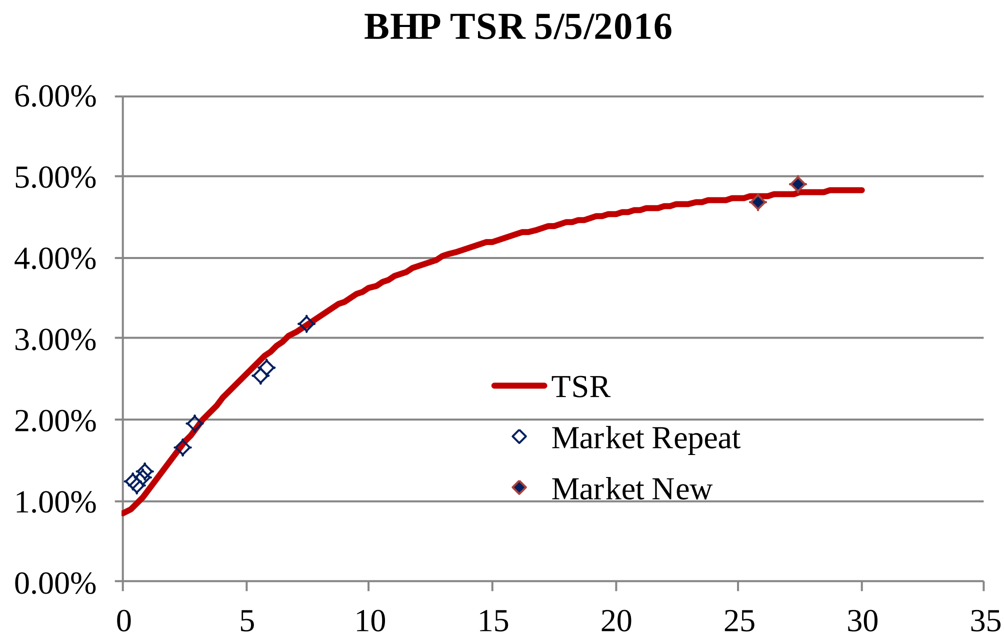


Figure A-8 - BHP Curve and Traded Securities Using 3 Components; 5/5/2016

The calculated TSR in Figure A-8 can’t be used for pricing. If we used the spreads that were calculated on 5/4/2016 for traded securities and applied them to the curve in Figure A-8, the resulting prices would be different from the market prices on 5/5/2016. There is no curve that can correctly replicate the price of all bonds that traded the previous day, but we can calculate a shifted curve that would correctly price the aggregate of all the bonds that traded on both days. The adjusted curve is shown in Figure A-9.

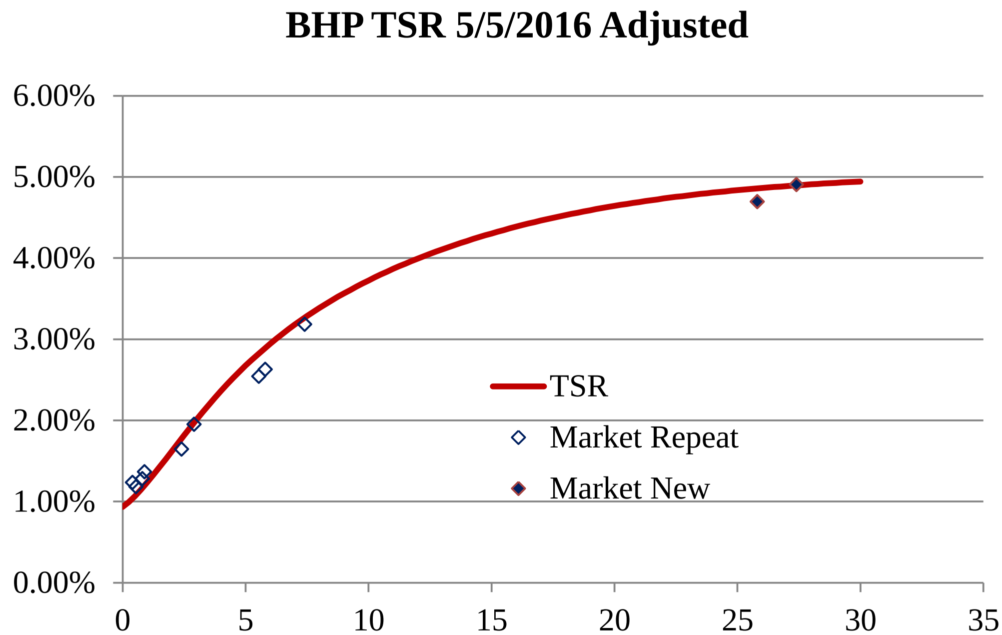
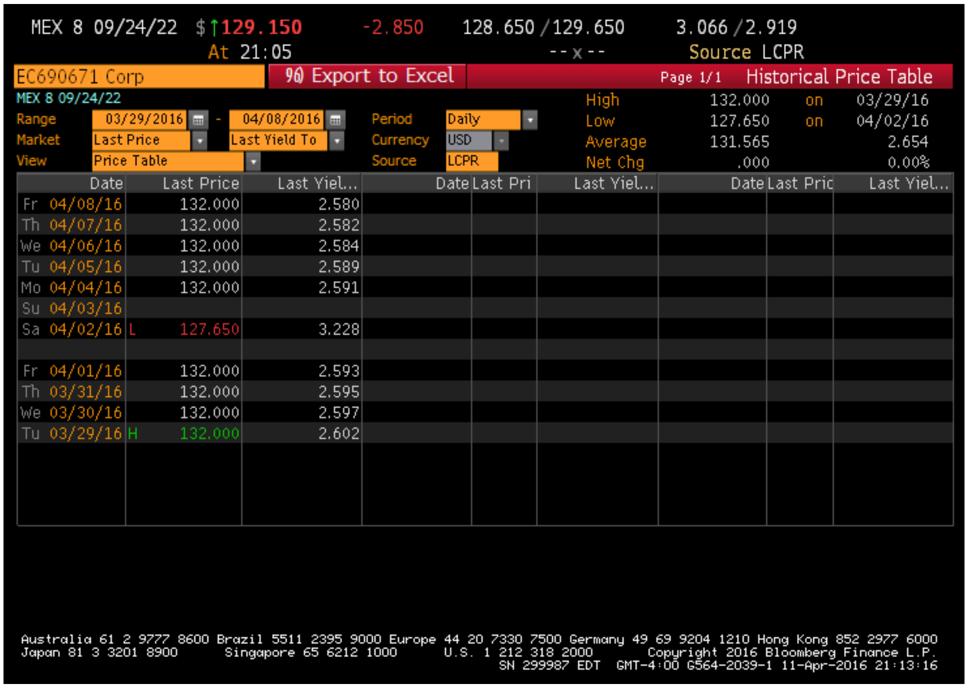


Figure A-9 - BHP Adjusted Curve and Traded Securities Using 3 Components; 5/5/2016

Comparing Figures A-9 and A-7, we can see that securities with a maturity of between 5 and 10 years are slightly below the curve and are positioned similarly relative to the curves. However, the same bonds are positioned closer to the curve in Figure A-8.

We can use the curve in Figure A-9 to price the bond with a maturity of 10 years in Figure A-7 that was not priced on 5/5/2016 after performing time series analysis, a subject that will be discussed in the following sections.

There are many corporate, emerging markets or sovereign securities that are not priced by most pricing sources. We can price those securities from their curves. Figure A-10 shows the daily price of one such security. Figure A-11 shows the intraday pricing of the same security. It is interesting to note that even though during the day it trades at various prices, it closes at a price of 132.



Date	Last Price	Last Yiel...	Date	Last Pri	Last Yiel...	Date	Last Pric	Last Yiel...
Fr 04/08/16	132.000	2.580						
Th 04/07/16	132.000	2.582						
We 04/06/16	132.000	2.584						
Tu 04/05/16	132.000	2.589						
Mo 04/04/16	132.000	2.591						
Su 04/03/16								
Sa 04/02/16 L	127.650	3.228						
Fr 04/01/16	132.000	2.593						
Th 03/31/16	132.000	2.595						
We 03/30/16	132.000	2.597						
Tu 03/29/16 H	132.000	2.602						

Figure A-10 - Historical price of a Mexican bond

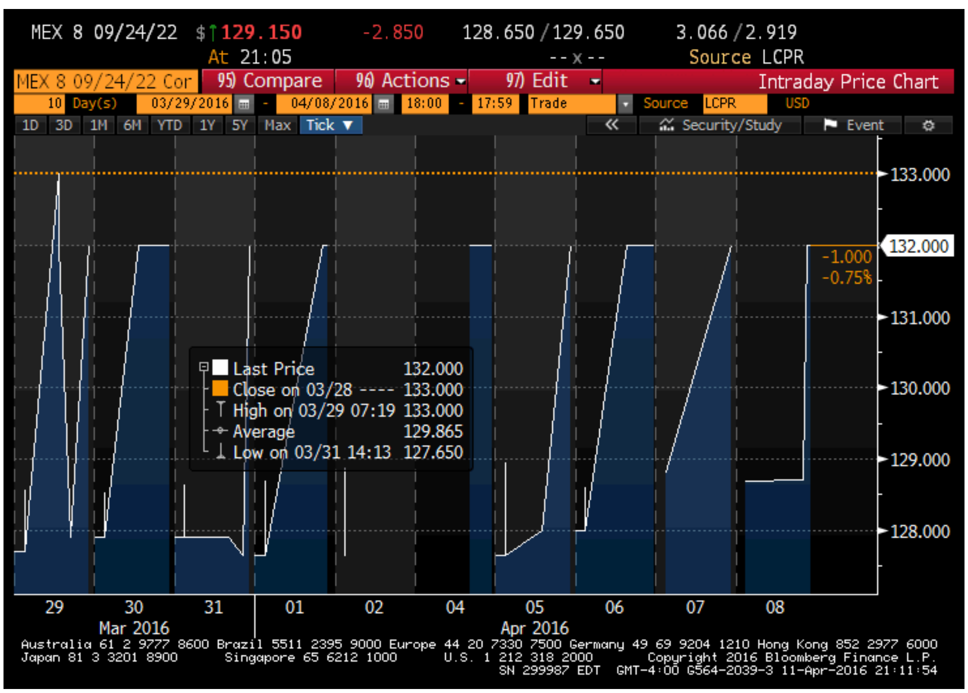


Figure A-11- Intraday price of Mexican bond

Callable Bonds

Valuation of the call value of a callable bond is a very complex business. Other than the model that was described in “The Advanced Fixed Income and Derivatives Management Guide” there are virtually no models that are both practical and accurate. Since nearly all callable bonds have a call option that can be exercised at any time after the first call date, their valuation is completely different from European options which can be exercised only on a single date. Even Bloomberg’s calculation of American option values is sometimes off by more than a factor of 2.

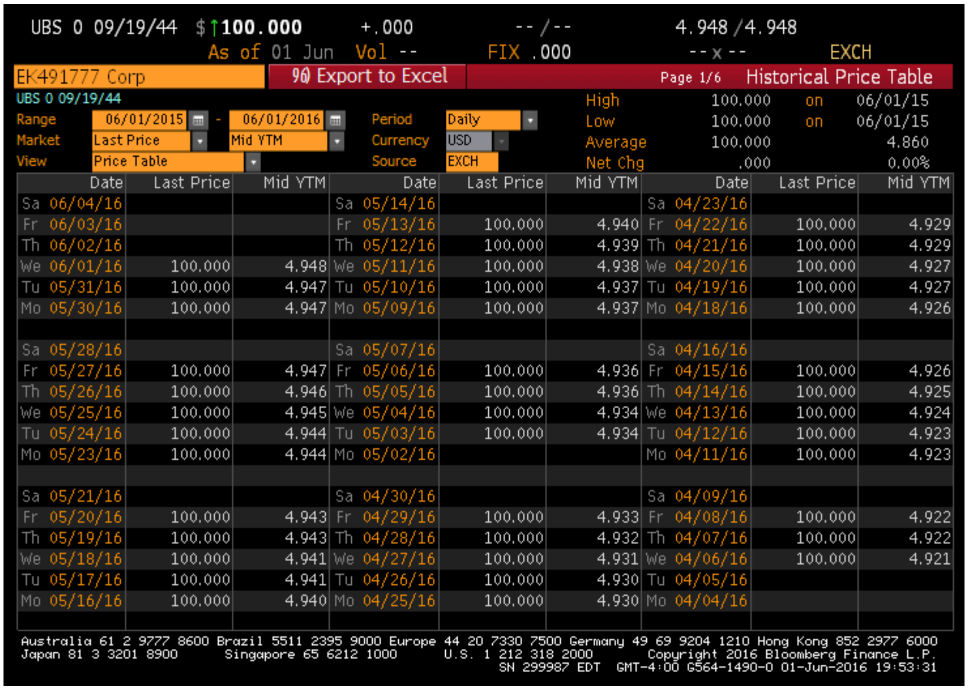
We calculate the Term Structure of Volatility (TSV) for countries where the data is available. Callable Agency bonds are priced by assuming a beta and correlation of one with the TSV. This is an accurate assumption, since Agencies often issue floating rate bonds with a spread of a plus or minus a few basis points to Libor.

For corporate bonds, we use a correlation of 0.5 and a spread beta of about 0.8 to estimate the value of the call option. Once the option value is calculated, we calculate the implied yield volatility that would price that option accurately and use the same volatility to calculate the value of the option on a following day, if the bond’s price needs to be calculated. The reason for this approach is that sometimes traded volatilities can have pricing errors. For example, in one instance the two year forward volatility of a three year bond which was 50% on one day, increased to 59% the following day. The call value of an out of money five year bond that was callable in two years, increased from 2 to about 5 due to the change in volatility. This would imply a three point drop in the value of the bond, however, the change in price was significantly below that. For this reason, the call value is calculated based on the implied volatility of the previous day.

Once we calculate the call value of a callable bond, we price an otherwise non-callable bond from its spread to curve and subtract the call value to calculate the price of the callable bond.

Without calculating the call value of a callable bond, there is no other way to calculate its price, if the bond has not been traded. Matrix pricing or interpolation can’t be used for pricing callable bonds, especially when the bond is trading close to the call price. For us, it is a puzzle on how some pricing services that don’t have good derivatives capabilities price callable securities.

There are many callable bonds that are either not priced or not priced correctly. Figure A-12 is the price table of a zero coupon bond that is not priced by Bloomberg service. This bond is callable by yield. The price table shows that for an entire year, the bond was priced at 100.


UBS 0 09/19/44 $↑100.000 +.000 -- / -- 4.948 /4.948
As of 01 Jun Vol -- FIX .000 -- x -- EXCH
EK491777 Corp 96) Export to Excel Page 1/6 Historical Price Table
UBS 0 09/19/44
Range 06/01/2015 - 06/01/2016 Period Daily
Market Last Price Mid YTM Currency USD
View Price Table Source EXCH
High 100.000 on 06/01/15
Low 100.000 on 06/01/15
Average 100.000 4.860
Net Chg .000 0.00%

Date	Last Price	Mid YTM	Date	Last Price	Mid YTM	Date	Last Price	Mid YTM
Sa 06/04/16			Sa 05/14/16			Sa 04/23/16		
Fr 06/03/16			Fr 05/13/16	100.000	4.940	Fr 04/22/16	100.000	4.929
Th 06/02/16			Th 05/12/16	100.000	4.939	Th 04/21/16	100.000	4.929
We 06/01/16	100.000	4.948	We 05/11/16	100.000	4.938	We 04/20/16	100.000	4.927
Tu 05/31/16	100.000	4.947	Tu 05/10/16	100.000	4.937	Tu 04/19/16	100.000	4.927
Mo 05/30/16	100.000	4.947	Mo 05/09/16	100.000	4.937	Mo 04/18/16	100.000	4.926
Sa 05/28/16			Sa 05/07/16			Sa 04/16/16		
Fr 05/27/16	100.000	4.947	Fr 05/06/16	100.000	4.936	Fr 04/15/16	100.000	4.926
Th 05/26/16	100.000	4.946	Th 05/05/16	100.000	4.936	Th 04/14/16	100.000	4.925
We 05/25/16	100.000	4.945	We 05/04/16	100.000	4.934	We 04/13/16	100.000	4.924
Tu 05/24/16	100.000	4.944	Tu 05/03/16	100.000	4.934	Tu 04/12/16	100.000	4.923
Mo 05/23/16	100.000	4.944	Mo 05/02/16			Mo 04/11/16	100.000	4.923
Sa 05/21/16			Sa 04/30/16			Sa 04/09/16		
Fr 05/20/16	100.000	4.943	Fr 04/29/16	100.000	4.933	Fr 04/08/16	100.000	4.922
Th 05/19/16	100.000	4.943	Th 04/28/16	100.000	4.932	Th 04/07/16	100.000	4.922
We 05/18/16	100.000	4.941	We 04/27/16	100.000	4.931	We 04/06/16	100.000	4.921
Tu 05/17/16	100.000	4.941	Tu 04/26/16	100.000	4.930	Tu 04/05/16		
Mo 05/16/16	100.000	4.940	Mo 04/25/16	100.000	4.930	Mo 04/04/16		

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Figure A-12- Zero Coupon Callable Bond; high and low prices are 100 for the year

We price such bonds by calculating the call value and subtracting it from the present value of the security, assuming that it not callable.

Bonds without a TSR

There are many issuers that have only one or two outstanding bonds with a given rating. For these securities, a TSR can’t be calculated. We price these bonds based on generic TSR’s that we produce from the aggregate of all TSR’s in a given currency. We segregate about 600-800 TSR’s that we calculate for US corporates on a daily basis, into 16 tranches, based on their yield levels, that we assign internal ratings from AA+ to CCC+ through a rigorous process that eliminates suspects TSR’s. The TSR for each tranche will be the average of about 30 corporate TSR’s; they represent our own internal rating based on market yields. We believe this is a better method of defining ratings than using rating agency values, since the markets quite often anticipate a rating change and price bonds accordingly. Moreover, bonds that have no ratings will be easily classified into one our tranches based on their yield or spread and maturity.

Every bond in the market will be assigned a rating and its spread relative to that rating is calculated every day. If the bonds of an issuer are not traded at all on a given day, we use our internal rating of that issuer to estimate its price change. This is a very safe method for pricing bonds that have not traded. In the event of a credit event the bonds will trade and this won’t be an issue.

Suppose that an issuer has two bonds X and Y with maturityies of 5 and 20 years and spreads of 20 and 25 bps relative to their internal BBB TSR on a given day. If on the following day, bond

X trades with a spread of 16 bps relative to the generic BBB, we then estimate that the spread of Y has contracted by 4 bps relative to the generic BBB and thus we price it accordingly. Note that the change in spread is relative to the curve, which itself is subject to a change in shape. For example, on a day that payroll data is weak, the yield curve can steepen and the yield difference between 5 and 20 year maturities increase by 10 bps. In our example, even though the yield of Y has contracted by 4 bps, the curve has steepened by 10 bps, resulting in a net increase in yield of Y relative to X of 10 bps. For a bond with a 20 year maturity and 12 year duration, the price impact would be 1.2%. With matrix pricing, we would not capture the 10 basis point change in the shape of the curve.

If a traded bond has a very short maturity, its yield adjustment can be very large an unrealistic; we ignore the yield change of bonds with a maturity of less than one year. For example, if the price of a bond that matures in 2 months changes by a bid-ask amount of 0.25, the implied yield shift will be 150 bps. Such a yield shift is not reasonable and can distort the price of long dated bonds by several points.

Likewise, we assign an internal TSR to all callable bonds. If the issuer of a callable bond has other bonds that are not callable, we use their generic TSR for valuation of the callable bond. If there are no non-callable bonds, we assign a generic TSR by iteration. We estimate the yield to worst and use that as a starting point to assign a generic TSR. We then calculate the option value and estimate the yield of non-callable security. From that yield we assign a new TSR and repeat this process.

Swaps and Libor

We calculate a TSR for Libor and Swaps for more than 30 countries. Our TSR for developed markets is very accurate and it is within 1-2 bps of the actual traded market. Through the Adjustment Table (AFIDMG) we replicate the market traded prices of all swaps and can use our TSR to price off-the-run swaps. Our model uses the smooth TSR instead of interpolation and we believe it to be the most accurate of all practical models in the market.

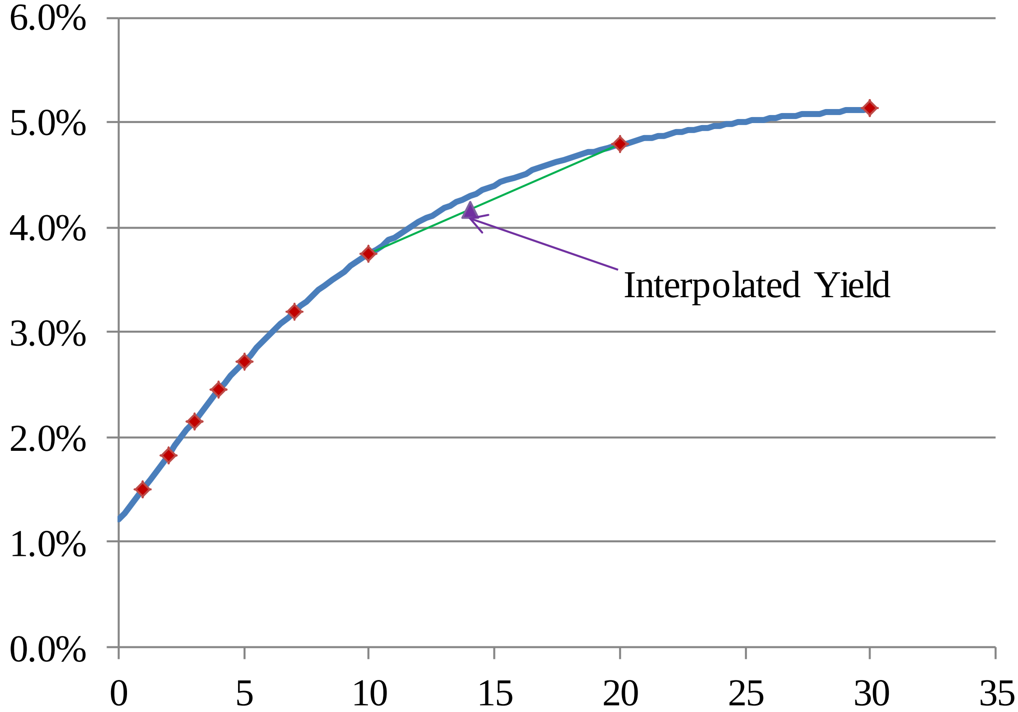

Figure A-13- Zero Coupon Swap Curve

In Figure A-13, the interpolated yield would be off by about13 basis points from the curve resulting in a pricing error of about 1.35%.

Swaptions and Bond Options

Bond options are completely different from equity options; unlike equities, bond prices have a maximum value. Close to maturity, bond prices approach par and option values fall; a forward bond has different durations and risks than a spot bond.

European bond options can only be exercised at expiration while American bond options can be exercised from a start date through an expiration date and they can be continuous or discrete (coupon payment dates). Callable bonds are usually like American bond options

Arbitrage free requirement imposes a very stringent constraint on the possible distribution of all outcomes in a way that guarantees the price of a call and a put are equal at option expiration for an at-the-money option. This is called call-put parity. However, this is true only at the expiration date and for the present time, the call-put parity is not applicable to fixed income options; an at the money forward call has to be discounted at a lower average rate than a forward put since the path of rates that will lead to a call imply lower forward discount rates while the opposite is true for a put option.

The most popular method of pricing the value of bond options is Black-76 which assumes that interest rate volatility is similar to equity volatility. Since price-yield relationship is not linear, Black-76 is not strictly arbitrage free. For short dated options, the pricing error is very small, but for long dated options and callable bonds, Black-76 is not accurate. Additionally, Black-76 assumes a constant volatility at all forward rates and there is no mechanism to discount call options at a lower rate than put options. Since 2008, the one year volatility of short rates has been more than double the volatility of long rates, thus the assumption of constant yield vaolatility has not been bourne out by the markets.

Monte-Carlo and binomial tree models converge very slowly with the square root of the number of steps and are not practical solutions for pricing bond options, but outside of our model, are the only methods that can discount the path of interest rates accurately.

We use a proprietary method with the accuracy of closed form solutions for bond options, if one existed. Unlike Black-76 we impose arbitrage free requirement on the price of the forward paths of interest rates and at every future exercise point we use a discount function that reflects the average path of interest rates to get to that point. Our volatility surface perfectly matches all volatilities at all forward rates exactly (with the market traded volatility).

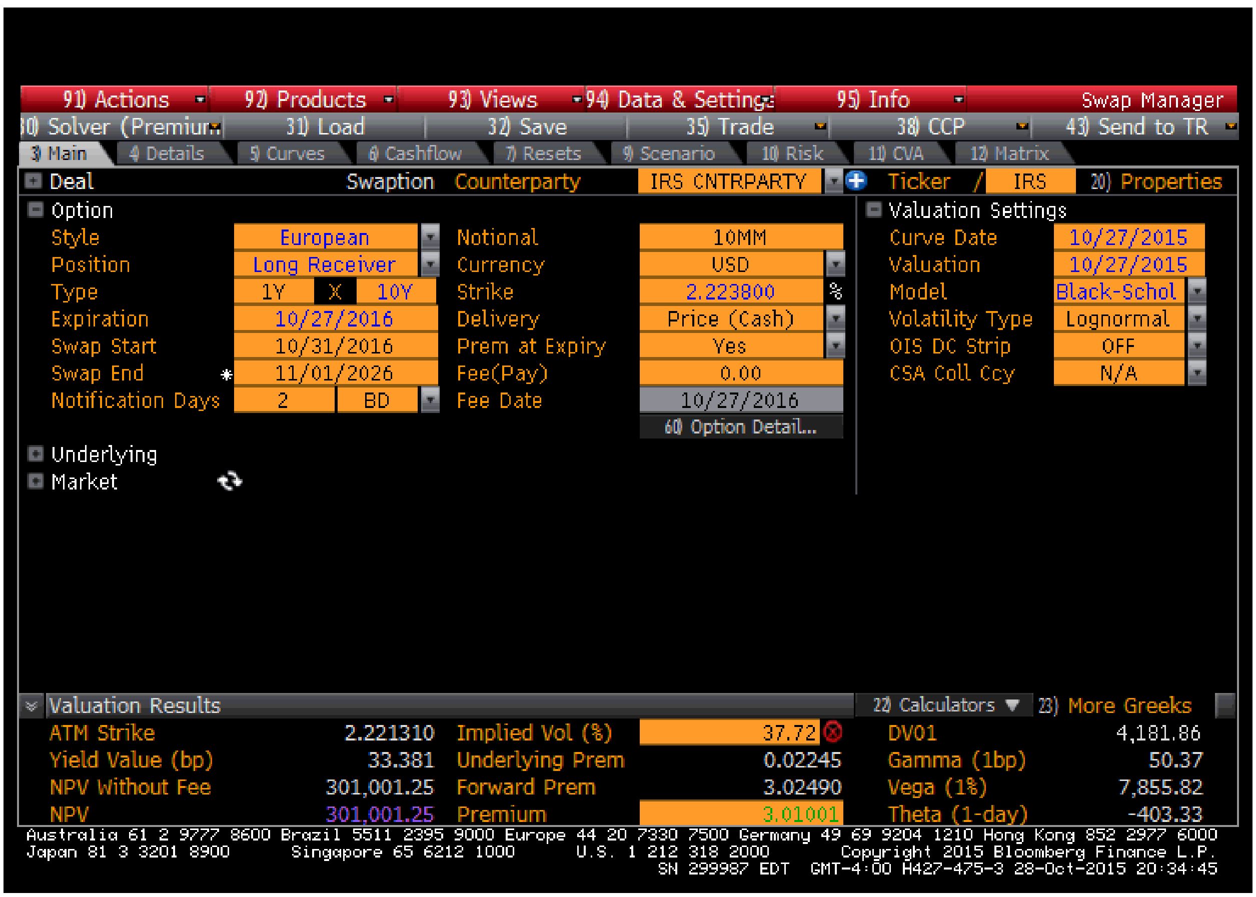

Figure A-14– Bloomberg European Call Price

Figure A-14 shows the calculated price of a European call swaption on 10/27/2015, for a 1 year x 10 year to be 3.01. Black-76 model yields a value of 2.95 and our model is very close at 2.933. Bloomberg price appears to be off, even though forward rates are the same. For longer dated options, where convexity becomes significant, Black-76 model pricing error is much higher. For a 5 year by 20 year forward call option Black-76 produces a price of 9.95 while our model is 9.53.

For American options, the universally accepted method of calculation is building a binomial tree. Binomial trees converge relatively slowly and have few nodes for early exercise, leading to relatively large pricing errors. Our proprietary method has the accuracy of closed form solutions for bond options if they existed. At every future exercise point we use a discount function that reflects the average path of interest rates to get to that point and our volatility surface perfectly matches all volatilities at all forward rates exactly (with the market traded volatility).

Figure A-15– Bloomberg American Call Price

Figure A-15 shows the calculated price of an American call swaption for the swap in Figure A-14. The calculated premium by Bloomberg is off by about 60%. Because the options are for forward at the money and the yield curve is steep, the forward implied coupon would be higher than the coupon of a10 year swap at the present. If the American call option is exercised immediately, the premium would be worth 1.73. However, given that the option premium is much higher, it does not make sense to exercise the option immediately. Suppose that there is 50% probability to exercise the option and we assume that there is a constant probability that the option will be exercised any time through the expiration. In this, on average the option will be exercised in six month and we will realize about 1.73 extra than a European option with a 50% probability for six months, i.e., 1.73/2 × 0.5=0.43. Thus, the premium should be worth about 0.43 over a European option. However, this amount is overstated, since the likelihood of an exercise in the first month is significantly less than the likelihood of an exercise in the last month. If we assume that the probability of exercise is not a constant, but rather is proportional to time, we can estimate the premium should be about 1.73/6=0.29 over European. Our model, provides a premium of 0.32 more for the American option vs. European option.

Table A-2 – Option Premiums: Our calculations and Bloomberg's

Parent				Option			
Cpn	**Maturity**	**Fwd**	**Mkt**	**C/P**	**A/E**	**Expiry**	**Strike**
2.2238	10/31/26	100	101.73	C	A	10/27/16	100
2.2238	10/31/26	100	101.73	C	E	10/27/16	100
2.2238	10/31/26	100	101.73	P	A	10/27/16	100
2.2238	10/31/26	100	101.73	P	E	10/27/16	100

Premium			Volatility		
Fair	**Model**	**Bloomberg**	**Mkt**	**Fair**	**Model**
3.252	3.252	5.288	0.377	0.377	0.379
2.933	2.933	3.01	0.377	0.377	0.379
2.939	2.939	7.645	0.377	0.377	0.379
2.925	2.925	3.018	0.377	0.377	0.379

The Bloomberg premium of the American put option is way off, considering that given the steep yield curve, the early exercise of the option is not likely and thus, the American put's price should be very close to the European Put.

Corporate Bond Options and Callable Bonds

We probably have the only process for pricing fixed rate and floating rate options of a credit security on a consistent basis. Corporate bond options require a correlation model and spread beta. Historical values for those can be used. Outside of Libor (swap) arbitrage free requirement, a second dimension for the arbitrage free forward distribution of credit rates consistent with the beta and correlation needs to be built. Only after building a three dimensional swap-credit-time volatility surface, can credit options be priced and we have such a process in place. For example, for a corporate issuer that has both fixed and floating rate callable bonds, we can price both options by using the same beta and correlation coefficient in an arbitrage free way. Outside of our model there is no other practical method for pricing floating and fixed rate bonds of an issuer in a consistent basis. Monte Carlo simulations in three dimensions converge way too slowly to be of practical use for pricing corporate options.

Other Options and Currencies

We price all currencies and currency forward buy building a TSR for currency forwards. From the TSR for currency forward, we can calculate the forward price of any currency relative to any other currency.

Options on currencies are generally the most diverse of all options and many exotic options trade primarily for currencies. We price exotic options using closed form solutions and we can relatively quickly model some of the most complex options on currencies and other assets. For example, a long dated option on a global stock index future, provided that EURUSD is below or above some level. Such an option is called a correlation option. The following list provides some of the options that we can price:

- Barrier Options
    - Single Barrier Options
        - Up and in call or put
        - Up and out call or put
        - Down and in call or put
        - Down and out call or put
    - Double Barrier
        - Touch and in call or put
        - Touch and out call or put
- Partial Time Barrier Options
    - Early Monitoring
        - Up and in call or put
        - Up and out call or put
        - Down and in call or put
        - Down and out call or put
    - Late Monitoring
        - Up and in call or put
        - Up and out call or put
        - Down and in call or put
        - Down and out call or put
- Look Options
    - Floating Strike Look Options

        - At expiration, the option holder has the right to exercise it at the extreme price that it traded during its life
        - Since the option is almost always exercised, the premium is very high
        - The strike price can be adjusted by a factor (f>=1 for calls, f<=1 for puts)
        - The monitoring period can be adjusted to make the option partial time floating strike look.
    - Partial Time Look Options
        - The holder has the right to exercise the option at the highest value during the monitoring period
    - Partial Time Look Barrier Options
        - If the barrier is not hit in the first monitoring period, a floating strike look option is issued
- Other Exotic Options
    - Soft Barrier Options
        - A soft barrier option is partially knocked in or out depending how much its price penetrates the gap between two barriers.
        - There can be up or down and in and out for call and puts
    - Digital (Binary) Barrier Options
        - Digital options pay a fixed amount if the option at expiration is in the money or nothing at all.
        - There are 16 option combinations (call/put, in/out, up/down, asset/cash)
        - There are 8 combinations for knock-ins (up/down, asset/cash, when hit/expiry)
        - There 4 combinations for knock-outs that are both put and call